Research Article

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Deflectometric measurement of large mirrors

Abstract: We discuss the inspection of large-sized, spherical mirror tiles by ‘Phase Measuring Deflectometry’ (PMD). About 10 000 of such mirror tiles, each satisfying strict requirements regarding the spatial extent of the point-spread-function (PSF), are planned to be installed on the Cherenkov Telescope Array (CTA), a future ground-based instrument to observe the sky in very high energy gamma-rays. Owing to their large radii of curvature of up to 60 m, a direct PSF measurement of these mirrors with concentric geometry requires large space. We present a PMD sensor with a footprint of only \(5 \times 2 \times 1.2 \text{ m}^3\) that overcomes this limitation. The sensor intrinsically acquires the surface slope; the shape data are calculated by integration. In this way, the PSF can be calculated for real case scenarios, e.g., when the light source is close to infinity and off-axis. The major challenge is the calibration of the PMD sensor, specifically because the PSF data have to be reconstructed from different camera views. The calibration of the setup is described, and measurements presented and compared to results obtained with the direct approach.

Keywords: 3D-metrology; deflectometry; interferometry; optical testing; telescope.

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1 Introduction

We present measurements of the surface structure and optical quality of large-sized spherical mirrors, e.g., as foreseen for use on the CTA [1], by Quantitative PMD. PMD is a novel tool to measure reflective optical surfaces with an accuracy in the sub-micron range and a local depth resolution in the nanometer regime [2–7]. Deflectometry is easy to apply as no null test geometry is necessary and the object under test does not have to be precisely aligned. This makes deflectometry competitive (compared to interferometry) for medium precision in-line inspection. It is already established, e.g., in the eye-glass industry, as well as in wafer- and car window inspection [8].

Deflectometry is a scalable technique, enabling quantitative surface inspection of objects from sub-mm size [9] to meter-size. Using PMD to measure the surface shape of large-sized mirrors for astronomical instrumentation is therefore an obvious application. As of today, the accuracy of deflectometry does not allow for the nanometer precision required for mirrors for optical or X-ray astronomy. However, the technique can be successfully applied to mirror tile prototypes [10, 11] developed for the CTA; the technique of measuring cosmic gamma-rays from ground does not call for diffraction limited imaging, but the PSF of each mirror must assemble 80% of the light intensity incident on the mirror surface from a light source at infinity, within a circle of <17 mm diameter. About 10 000 spherical mirrors of hexagonal shape have to be measured, with dimensions 0.78–1.5 m (flat-to-flat) and radii between 11 m and 60 m, depending on the telescope type. Here we report the measurement of a 1.2 m flat-to-flat prototype mirror with radius of curvature 32 m.

What requirements are there?

- Because of the large number of mirrors, easy handling without precise positioning is necessary.
- The PSF should be obtained for a light source at infinite distance (the Cherenkov radiation source is at approximately 10 000 m height). On the telescopes, the mirrors will be partially used in an off-axis orientation.
- It would be beneficial to measure not only the PSF shape and extension, but to get a map of the mirror surface, as this would help spotting problems during the manufacturing process.

The standard method, so far, is a PSF measurement in a concentric configuration. A small light source is positioned in the center of curvature and the PSF is easily acquired at a close-by position. The method is simple and accurate, but
has some drawbacks: first, depending on the type of mirror, it requires a large lab with more than 60 m length. This excludes measurements in a climate chamber e.g., as used to characterize the mirrors under different environmental conditions; second, the light source is not at infinity; third, no local information on the mirror surface is available.

Deflectometry instead is simple and reliable when used in a (space requiring) concentric geometry. But deflectometry does not require a null-test geometry, which is its great advantage over interferometry, so deflectometry can be adapted for short distance measurements as well. However, there are challenges to overcome for the large objects under study: The short distance sensor displays large deflection angles, which requires sophisticated calibration; the more so as we put forward a multi-camera measurement with stitching of the mirror surface. (Principally, a pattern generator (TV screen) is required twice as large as the object under test, but such a screen is not available for reasonable cost). In the following, we will explain how we solved the challenges mentioned above. We will present a PMD sensor with a footprint of $5 \times 2 \times 1.2$ m$^3$ that is able to measure the PSF of CTA mirrors to an accuracy that satisfies mirror mass inspection. In particular, we will present measurements of a reference mirror with a height accuracy of better than 10 μm.

2 Phase measuring deflectometry

We will briefly describe quantitative phase measuring deflectometry, invented [2, 12] and developed at the Institute of Optics, Information and Photonics, see, e.g., [3, 7]. The measurement principle is simple: a camera observes the mirror image of a sinusoidal pattern generated by a screen, and reflected by the specular surface of the object under test (Figure 1). The mirror image displays distorted fringes which encode the local object slope.

Following the path of the light rays from the light source to the camera, including the reflection at the surface, the normals of the object surface can be reconstructed. For quantitative measurements, the setup has to be precisely calibrated. Calibration is the crucial issue of quantitative PMD.

2.1 Calibration

The calibration requires three steps:

- internal camera calibration: we use a standard bundle adjustment with an extended pinhole model [13]. Model free approaches are possible as well [14].

- screen calibration: the position of each pixel is evaluated via photogrammetry.

- geometric calibration: screen, object and camera must be localized in a global coordinate system.

The last part is the most difficult step in the calibration procedure because the camera does not observe the screen directly as it is only visible via the reflection at the object (Figure 2). Unfortunately, the geometric calibration is also the most critical step concerning the global accuracy. Within the last years we developed a simplified, user friendly, calibration that is achieved by performing several measurements of a specular object (e.g., a planar mirror or a sphere, convex or concave), at different tilted positions [15, 16]. No precise positioning of the object is necessary. Not only are the camera and the screen position found, but the object position can be calculated as well. This is done within one global optimization step, where the distances of the measured points on the screen to the reflected rays of view of the camera are minimized. The solution is unique apart from a scaling factor. To calculate that factor, a known radius of the calibration sphere or the
known size of the screen can be used. The described calibration method had to be adapted for the measurement of large mirrors, as explained below.

2.2 Height ambiguity

PMD measures primarily the local slope of the specular surface by observing the deflection of light rays. By following the path of light rays from the light source via the object to the camera, the spatially resolved gradient (the normals) of the object can be reconstructed. This implies – a paradox – that the object shape and its position must already be known, as a false assumption on the position of the object will lead to false normals (as sketched in Figure 3). For common applications our solution to solve this ‘ambiguity problem’ is the use of a second camera (stereo deflectometry [12]).

3 Measurement objectives

Figure 4 displays the prototype CTA mirror used for the PMD measurements presented in this paper. The CTA mirrors are concave spherical composite mirrors, manufactured from lightweight materials to keep the telescope mass low. A typical approach is to glue onto a honeycomb substrate a thin glass sheet, which is later equipped with either an aluminum/quartz or dielectric coating [10, 11]. CTA mirrors are hexagonal in shape, with a diameter of 0.78–1.5 m (flat-to-flat) and radii of curvature from 11 m up to 60 m (depending on the telescope type). One major quality criterion is the spatial extent of the PSF, which must stay within specifications at temperatures between -15°C and +25°C, even after extensive temperature cycling. A possibility to measure the PSF in a climate chamber at various temperatures would therefore be beneficial.

3.1 Classical 2f-setup

The common, simple (null test) solution to measure the PSF is to place a point source at the center of curvature (at the 2f-distance). Since the mirrors are spherical, the light incident on the mirror surface is reflected back into a PSF close by the source position (see Figure 5). The PSF is captured and its size can be calculated. Due to the large mirror radii, the 2f-technique requires a large laboratory. This is a real drawback, the more so, as quality tests in a climate chamber are required during mirror development. Furthermore, the measurement requires

Figure 4 CTA mirror sample used for this work: the mirror is hexagonal in shape, with dimensions 1.2 m (flat-to-flat) and a radius of curvature of about 32 m.

Figure 5 Sketch of the classical 2f-setup.
As a detector, either a photodiode mounted on a scan-table is used or the PSF is imaged on a screen and photographed by a CCD camera.
a time consuming alignment of the mirrors of ≈10 min, which is also a practical disadvantage. The measurements are performed at different distances, which takes ≈5 min. The evaluation of the captured images to calculate the PSF at the optimum distance still needs about 1 h. Since the project is still in development there is room for improvement. Note that the 2f-technique principally cannot deliver the PSF at the focal point (1f-PSF). PMD has the potential to overcome these problems as the PSF can be calculated from the surface data for any incident light distribution.

### 3.2 Long working distance (LWD) PMD setup

A concentric PMD setup at 2f-distance (see Figure 6) is simple and robust because only one camera and a (small) TFT-monitor is sufficient to measure the entire mirror (see Figure 7). Monitor and camera are placed at roughly the center of curvature. (A similar setup was presented by Su [17].) As in the classical 2f-setup, the LWD setup requires a large laboratory and the same time and effort for the alignment as for the 2f-method, but provides slope information for the mirror surface. The measurement takes about 1 min and the evaluation of the acquired data approximately 5 min, with room for improvement.

The calculation of the slope data is robust against calibration errors because the reflection angles, which must be reconstructed during the evaluation, are very small (null test geometry) and the influence of the position assumption that afflicts the global accuracy is minimized. The PSF is not directly acquired but calculated by ray tracing from the surface data, allowing calculation of the PSF for real-case scenarios, e.g., when the light source is off-axis or at a distance close to infinity. As an example, Figure 8 shows a comparison of the PSF of a CTA prototype mirror acquired from the LWD-PMD (left) and a directly captured PSF using the classical 2f-setup (right).

![Figure 6](image)

**Figure 6** Left: Image of the concentric LWD PMD setup. One camera and a small screen are sufficient to measure the entire mirror. The second (slave) camera is for the stereo method. Right: a sketch of the measurement principle.

![Figure 7](image)

**Figure 7** Image seen by the camera of the LWD PMD. The camera is focused at the mirror, which reflects a sinusoidal pattern generated by the TV screen. Already here local details introduced by the mounting pads at the backside of the mirror are visible.

![Figure 8](image)

**Figure 8** Comparison of the PSF of a CTA prototype mirror, calculated from the LWD-PMD measurement (left) and directly measured using the classical 2f-setup (right). Note that the images display the PSFs not at the optimum 2f-distance (where the PSFs are smallest), but at a distance shifted by 30 cm to allow for a better comparison of the PSF structure.

### 3.3 Short working distance (SWD) PMD setup

Because measurements in a climate chamber are beneficial during mirror prototyping, and a fast (no alignment) quality check of a large number of mirrors is required, we implemented a small footprint PMD (SWD-PMD) which fits into an existing climate chamber, and allows for a simple placement of the mirror in the setup which can be done within several seconds as no alignment is necessary, (see Figure 9).

![Figure 9](image)

Because the measurement field is mainly limited by the size of the screen, the setup hosts four cameras. While the cameras’ fields of view cover the entire mirror area, they only observe a fraction of the screen pattern reflected by the mirror (see Figure 10). Hence, to measure the entire mirror, the separate measurements of the four cameras have to be stitched. The measurements of the four cameras are performed simultaneously. Compared to the LWD setup the measurement time increases because...
4 Results

For a full-field map of the mirror, four individual maps delivered by four cameras have to be combined. Discontinuities at the edges of the maps can only be avoided with an optimally calibrated system and an optimized evaluation method.

4.1 Calibration of the SWD setup

How to calibrate such a large setup? As mentioned, the internal camera calibration is standard. More difficult is the geometric calibration, i.e., the localization of the screen, the cameras and the object. For small objects, the geometric calibration can be performed by several measurements of a specular gauge object in different tilted positions. However, gauges, large enough to fill the measurement field for the CTA mirrors, are not available. Our workaround is to replace the gauge by one of the (very precise) CTA mirrors for the calibration. We have to accept that the mirror shape may change slightly, owing to gravity, when tilting the mirror, possibly leading to a calibration that is not perfect. So we first have to accept measurements that deliver only a first approximation of the real surface slopes of the object. In the following chapter we will explain how to get a second, better iteration.

4.2 Evaluation

In addition to the calibration problems, the evaluation of the measurements is more difficult than in small-scale PMD systems. For standard PMD we use the stereo method to solve the height ambiguity problem (see section 2.2). For the SWD PMD, the stereo principle cannot be applied, due to the small overlap areas. Furthermore, the standard evaluation does not calculate the object position with the...
accuracy sufficient for perfect stitching. Unfortunately, classical registration methods cannot be applied as well, because the objects display no salient features (neither at the height map nor at the slope map). It was therefore necessary to develop a new evaluation method. With the two known boundary conditions:
– the object is a sphere (but with unknown radius),
– for each camera view, the sphere is located at the same position in the global coordinate system,

we can find one optimal object position and simultaneously optimal camera positions (fine calibration). The knowledge about the object position solves the ambiguity and the stitching problem. With the known height values and the optimized camera positions, we can precisely evaluate the slope. Due to the fact that all calculations are done in one coordinate system, the data of the different camera views fit automatically and can be combined simply by averaging the slope data in the overlapping areas.

Figures 11 and 12 display the height and slope maps of an SWD-PMD measurement of the reference mirror before and after fine calibration, respectively. Despite of the large radius of curvature, the PSF in 2f distance is only about 10 mm. This justifies the use of this mirror as a reference to quantify the accuracy of our measurements. Figure 11 displays the accuracy before the fine calibration. The yet to be optimized calibration does not allow for a precise stitching without discontinuities. Eventually, after the fine calibration, smooth height and slope maps can be calculated (see Figure 12). Because the real surface error of the mirror is unknown, we assume the remaining global height deviation being caused completely by the sensor. This global accuracy is 10 µm on a measurement field with a diameter of ≈1 m, or a fraction of 10⁻⁵ of the mirror diameter.

5 Prototype measurement

After verification of the calibration, and the evaluation method as described above, we measure our CTA prototype mirror, shown in Figure 4. The height and slope maps are displayed in Figure 13.

We calculated the PSF and d80 diameter of the prototype CTA mirror from the height and slope maps, for a point source located at the optimum center of curvature. Table 1 shows images of the PSF for all three methods. Both the d80 diameter of the LWD and SWD measurements are
in agreement with the results obtained with the 2f-setup (note that the classical 2f-measurement setup uses a light source of about 2.5 mm diameter, resulting in a larger PSF), and very well within the required mirror PSF size tolerance, $d_{80} < 34$ mm, for 2f distance.

The deviation of the $d_{80}$ diameter measured with the SWD setup to the LWD gold standard is only 1.5 mm. This corresponds to a slope deviation of $< \pm 3$ arcsec. The deviation of the radius of curvature (LWD gold standard − SWD) is 8 cm or 0.25%. Part of the slight differences may be caused by a mechanical instability of the mirror that is measured in different orientations (horizontal axis for LWD-PMD and 2f, and vertical axis for SWD-PMD), but as well by the remaining calibration errors of the SWD PMD setup.

While studies about the long-term stability of the calibration are beyond the scope of this paper (and will be published elsewhere), we note that during the last months we measured different types of CTA prototype mirrors with the direct 2f-method, the LWD and the SWD PMD setup. As a result, the 2f-method (after correction for the extension of the light source) and LWD PMD measurements display good agreement and can therefore be considered as gold standard. Typical differences in the radius of curvature are in the range of $\pm 30$ mm ($\pm 0.9\%$), the diameter of the $d_{80}$ circle at the 2f-distance varies by $< \pm 1$ mm.

The SWD measurements display slightly bigger differences: the radius of curvature for different prototypes differs by $< \pm 120$ mm ($\pm 0.4\%$) against the gold standard and the diameter of the $d_{80}$ circle differs by $< \pm 2.8$ mm.

The difference of the measurement results between the SWD method and the 2f and LWD method is sufficiently small to satisfy the CTA specifications. The even better agreement between the 2f- and LWD results may be at least partly explained by the gravitational deformation of the mirrors under the influence of gravity, as argued above.

### Table 1

<table>
<thead>
<tr>
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<th>2f-Method</th>
<th>LWD-PMD</th>
<th>SWD-PMD</th>
</tr>
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<tbody>
<tr>
<td>PSF</td>
<td>[Image]</td>
<td>[Image]</td>
<td>[Image]</td>
</tr>
<tr>
<td>$d_{80}$ diameter (mm)</td>
<td>12.1</td>
<td>9.6</td>
<td>11.1</td>
</tr>
<tr>
<td>Radius of curvature (m)</td>
<td>32.03</td>
<td>32.06</td>
<td>32.14</td>
</tr>
</tbody>
</table>

The data are calculated at the distance of the minimum $d_{80}$ diameter. This $d_{80}$ diameter is displayed in the second row. The ‘optimal’ distance is displayed in the third row. Note that the 2f PSF is slightly enlarged because the light source is not a point source.
slop uncertainties of only a few arcsec. As we have shown, even the short working distance PMD, with its big deflection angles, satisfies these requirements. The measurement is quite simple; no precise positioning of the mirrors is necessary. The method is completely incoherent; it is therefore not prone to coherent noise or vibration artifacts. The deep reason for the simplicity of deflectometry – compared to interferometry – is that there is no requirement of a null-test geometry and there is no retrac error.

These advantages, however, can only be exploited after a sophisticated calibration. We re-consider that the requirements for the slope precision are in the arcsec range, and the global height accuracy is better than a few micrometers.

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References


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Christopher van Eldik received his PhD from Dortmund University in 2004, where he worked on the production of vector mesons in proton-nucleus collisions. After a short postdoc at DESY (Hamburg, Germany) he changed fields from particle physics to gamma-ray astronomy and became senior postdoc in the group of Werner Hofmann at Max-Planck-Institute for Nuclear Physics (Heidelberg, Germany). In 2011 he was appointed Professor of Physics (experimental Astroparticle Physics) at University of Erlangen-Nuremberg.

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